“A New Radiation Hypothesis”
by Max Planck
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Gentlemen! Fully ten years ago I had the honor of lecturing here on the foundations of a theory of heat radiation, one of whose essential assumptions is that, in the case of the generation of heat rays, a characteristic role is played by certain finite, indivisible quanta of energy, or elements of energy, of the size $\varepsilon = h\nu$, where h is the elementary quantum of action, 6.55×10^{-27} erg sec (M. Planck, *Verh. d. D. Phys. Ges.* **2**, 237, 1900; in altered form *Ann. D. Phys.* (4) **4**, 553, 1901).

As peculiar as this assumption is, when contrasted with the well-known and established presentations of electrodynamics and the theory of electrons, yet so many consequences follow from it, not only for the laws of black-body radiation but also for the elementary quanta of electricity and matter, and also, thanks to the researches of A. Einstein and W. Nernst, for the well-established specific heats of solids and liquids, that it appears quite justified to proceed further along the path already laid down and to lift the veil which still lies over the quanta of energy.

Of course, from the very beginning I have unceasingly worked to elaborate the conceptions of the processes of absorption and emission of heat radiation, but unfortunately without significant success. Difficulties arose from many sides – difficulties whose significance one may appreciate when one considers that even the validity of the fundamental equations of Maxwell-Hertz electrodynamics was brought into doubt, according to which any local electrodynamic disturbance is propagated as a spherical wave in all directions. In my opinion, however, one need not now go that far but should, instead, not flinch even before risky hypotheses, so that one can live with Maxwellian electrodynamics, which is so well established by the most precise optical measurements.

Such considerations encourage my reporting to you now on a new radiation hypothesis. I have developed it partly in response to criticisms of my theory by other researchers, of which the most recent is that of H. A. Lorentz (*Phys. ZS.* **11**, 1248, 1910), and I ask you to consider this hypothesis, which, I believe, may be rather fruitful.

For greater clarity, allow me first to review the conceptual development of my theory heretofore. I have assumed linear Hertzian oscillators as the centers for the absorption and emission of radiant heat. The excitation of such an oscillator with characteristic frequency ν is produced by that component $\mathfrak{E}_z$ of an incident electric field which lies along the oscillator’s axis. Namely, if J is the time-average of the square of $\mathfrak{E}_z$ and if we decompose J into its Fourier spectrum

$$J = \int_0^\infty \mathfrak{J}_\nu d\nu$$

then the quantity $\mathfrak{J}_\nu$ (which I have called the intensity of the vibration exciting the oscillator) yields the energy absorbed by the oscillator in the time *dt*:

$$\frac{3c^3\sigma}{16\pi^2\nu}\,\mathfrak{J}_\nu dt \qquad 1)$$

where *c* is the speed of light and σ is a small constant, namely the logarithmic decrement, due to damping, of the amplitude of the vibration of the oscillator.

In the case of isotropic, stationary black-body radiation, the spatial density $\mathfrak{u}_\nu$ of the frequency ν depends upon $\mathfrak{J}_\nu$ according to the relation

$$\mathfrak{u}_\nu = \frac{3}{4\pi}\,\mathfrak{J}_\nu\ . \qquad 2)$$

On the other hand, the energy emitted by the Hertzian oscillator in the time *dt* is

$$2\sigma\nu U dt$$

where *U* is the vibrational energy of the oscillator.

In a field of black-body radiation, the energy absorbed is equal to the energy emitted, hence

$$\mathfrak{u}_\nu \;=\; \frac{3}{4\pi}\,\mathfrak{J}_\nu \;=\; \frac{8\pi\nu^2}{c^3}\,U\ . \qquad 4)$$

In order to proceed from this equation to the laws of black-body radiation, we require the concept of temperature. This can be obtained from the general thermodynamic relation among temperature *T*, energy *U*, and entropy *S*

$$\frac{1}{T} \;=\; \frac{dS}{dU} \qquad 5)$$

in combination with the equally general relation between energy and probability

$$S = k\ \ln W \qquad 6)$$

where *W* is the probability that the oscillator will possess energy *U* and where *k* is 1.346×10^{-16} ergs per degree.

According to this, the problem comes down to calculating the probability that an oscillator of frequency ν would have a given energy *U*. I attempted to solve this problem by conceiving of *U* as a statistical average and I investigated the distribution of a very large quantum of energy *NU* among *N* identical oscillators. In order to arrive at a definite, finite value for this probability, I considered *NU* as the sum of a large number of identical, indivisible elements of energy of size $\varepsilon = h\nu$, hence

$$NU = P\varepsilon \,, \tag{7}$$

and I assumed that, for each possible distribution, or complexion, a definite number of elements of energy (possibly none) would fall to each oscillator. Letting W_N denote the number of all possible distinct complexions, we have

$$W_N = \frac{(N+P)!}{N!P!}$$

and the corresponding entropy

$$S_N = k \ln W_N$$

and so the corresponding entropy for a single oscillator is

$$S = \frac{S_N}{N} = k \left\{ \left(1 + \frac{P}{N}\right)\ln\left(1 + \frac{P}{N}\right) - \frac{P}{N} \ln \frac{P}{N} \right\} \tag{9}$$

from which, by Equation 7, we get

$$S = k \left\{ \left(1 + \frac{U}{h\nu}\right)\ln\left(1 + \frac{U}{h\nu}\right) - \frac{U}{h\nu} \ln \frac{U}{h\nu} \right\} \tag{10}$$

and finally, by substituting into 5), we get

$$U = \frac{h\nu}{e^{h\nu/kt} - 1} \tag{1}$$

for the energy of the oscillator, from which, by 4), we get

$$\mathfrak{u}_\nu = \frac{3}{4\pi}\,\mathfrak{J}_\nu = \frac{8\pi h\nu^3}{c^3}\,\frac{1}{e^{h\nu/kt} - 1} \tag{4}$$

for the spatial density of the black-body radiation.

The derivation above would naturally be immediately intelligible if we assumed that the actual energy U of each oscillator were, at each moment, an integral multiple of ε and therefore could change only by discrete amounts. I have attempted to elaborate this assumption further and even a year ago I expressed the hope that it could be accomplished. However, weighty misgivings came to the fore. One of the most difficult questions is, “How can such an oscillator absorb an energy element ε if it is hit by a heat ray?” It must absorb it from the incident exciting ray, and indeed suddenly and completely. Therefore, if the exciting ray, which could have an arbitrarily small intensity, is too small, then it could not be absorbed at all. This leads to the idea that for the oscillator a certain threshold exists, below which it is capable of no excitation at all, and above which the absorption begins with a whole element of energy. Moreover, as I belatedly emphasize here, M. Reinganum (*Phys. ZS.* **10**, 351, 1909) has already come upon the idea of such a threshold in his oscillator model.

However, the difficulties are not thereby removed. For, the taking up of a finite quantum of energy from a finite intensity of radiation can occur only in a finite time, which will be all the longer, the smaller is the intensity of the exciting vibration $\mathfrak{J}$ when compared with the quantum ε of energy. Now, the quantum of energy $\varepsilon = h\nu$ becomes larger with the frequency, whereas, on the other hand, the intensity $\mathfrak{J}$ falls off so rapidly that, for short waves, the time mentioned above must ultimately become immense. And this contradicts the assumption made; for if the oscillator has begun to absorb energy and if the incident radiation should suddenly cease, then the oscillator would be prevented from taking up the complete quantum of energy which it requires from time to time for the production of the statistical mean value U.

In my opinion, these considerations lead us to regard the absorption as proceeding completely continuously and, correspondingly, to regard the expression 1) for the energy absorbed as exact.

With that we remove the assumption of the absolute discontinuity of the energy U of the oscillator, and U need not be only an integral multiple of the quantum ε but can assume any value between zero and infinity. At the same time the thought of connecting probability with the absorbed energy becomes irrelevant. Instead, the value of the absorbed energy is immediately given by Equation 1).

In addition, the hypothesis is suggested that the emission of energy from the oscillator, on the other hand, occurs in jumps, according to the energy quanta and the laws of chance, quite independently of any simultaneous absorption. The emission of energy proceeds spontaneously, in determined quanta of size $\varepsilon = h\nu$, and the probability that an oscillator of characteristic frequency ν will emit an elementary quantum of energy in the time dt is equal to

$$\eta n dt \qquad 13)$$

where η is a constant (to be determined shortly) depending only on the nature of the oscillator, and where n is the number of whole energy elements which the oscillator possesses (i.e., n is that nonnegative integer for which

$$\frac{U}{\varepsilon} - n$$

is a proper positive fraction). Then we can write

$$U = n\varepsilon + \rho \qquad 14)$$

where $0 < \rho < \varepsilon$.

For example, if U is smaller than ε , then $n = 0$ and the oscillator will emit nothing at all. On the other hand, if U is large, we can neglect ρ in comparison with $n\varepsilon$ and regard the emitted energy as proportional to U, as was done earlier.

We next investigate the stationary state of vibration for the oscillator when it is in the field of black-body radiation. In that case, we cannot set the energy absorbed in the time dt equal to the energy emitted in that same span of time, for the former is continuous and the latter is discontinuous. In fact, the equilibrium is a statistical one and relates to the average values of the absorbed and emitted energies over long times. Under this assumption, we can write the condition for the stationary state, in obvious notation, as

$$\frac{3c^3\sigma}{16\pi^2\nu}\,\mathfrak{J}_\nu = \eta\bar{n}\varepsilon = \eta(\bar{U} - \bar{\rho})$$

The mean value $\bar{\rho}$ is clearly $\frac{\varepsilon}{2}$ and therefore

$$\mathfrak{J}_\nu = \frac{16\pi^2\nu\eta}{3c^3\sigma}(\bar{U} - \frac{\varepsilon}{2})$$

Since for large U this last equation must agree with 4), it follows that the emission coefficient η is

$$\eta = 2\sigma\nu$$

and the previous equation, together with 2), yields

$$\mathfrak{u}_\nu = \frac{3}{4\pi}\mathfrak{J}_\nu = \frac{8\pi\nu^2}{c^3}(\bar{U} - \frac{h\nu}{2})$$

in noticeable contrast to 4).

Now we consider again the determination of the temperature. For this we proceed just as we did above; i.e., we use the general thermodynamic equations 5) and 6) and ask for the probability that the oscillator will possess mean energy $\bar{U}$. We will get this probability by considering again the distribution of a very large quantum NU of energy among N identical oscillators. But now, in contradistinction to the earlier considerations, the energy U of an oscillator may possess values other than a whole multiple of ε. For the energy U of an oscillator at any given time t is determined uniquely from its energy U_0 at time $t = 0$ and the energy that it has absorbed and emitted in the span of time t. Moreover, for sufficiently large t, the initial energy U_0 becomes irrelevant to the determination of the probability of the energy U and can therefore be given an arbitrarily fixed value. Likewise, the absorbed energy is completely determined by 1) and is the same for all oscillators in the field of black-body radiation. (Spatial and temporal fluctuations of the intensity of the exciting radiation will be present but will have no influence, as a little thought shows.) Therefore considerations of probability relate only to the emitted energy, and this is, by our hypothesis, a whole multiple of ε. Hence, in the expressions 14) for the energies of the N oscillators, namely,

$$U_1 = n_1\varepsilon + \rho_1,\ U_2 = n_2\varepsilon + \rho_2,\ \ldots$$

it is only the integers n_1, n_2,... that are to be subjected to considerations of probability. But since the total energy

$$U_1 + U_2 + \ldots = N\bar{U}$$

is given, then so is given

$$P = n_1 + n_2 \; ... = \frac{(U_1 + U_2 + ...) - (\rho_1 + \rho_2 + ...)}{\varepsilon}$$

i.e.,

$$P = \frac{N(U - \varepsilon/2)}{\varepsilon} \qquad 17)$$

and it comes down to finding the distribution of a large number *P* of energy elements among *N* identical oscillators, just as before.